Цифровой рекурсивный формирователь отсчетов сигнала с линейной частотной модуляцией

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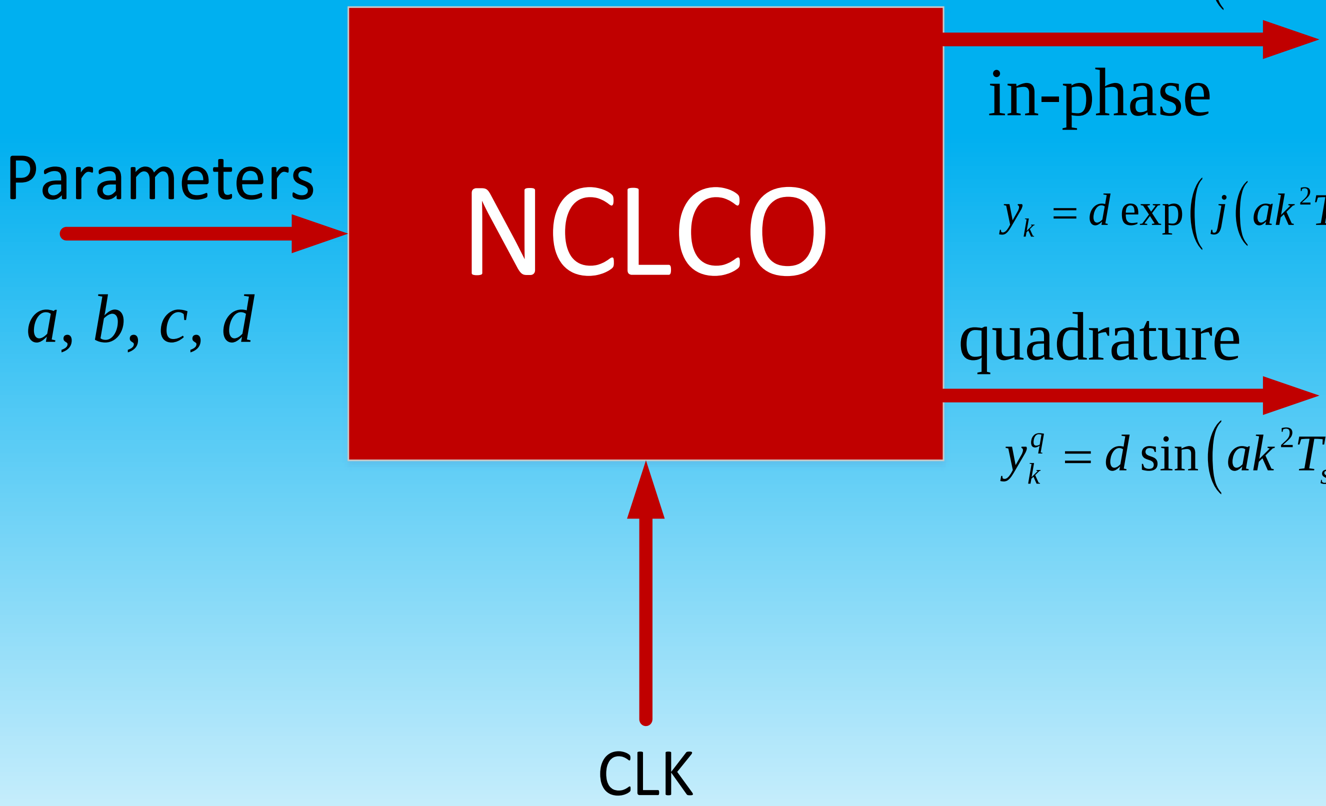
Numerically Controlled Linear Chirp Oscillator

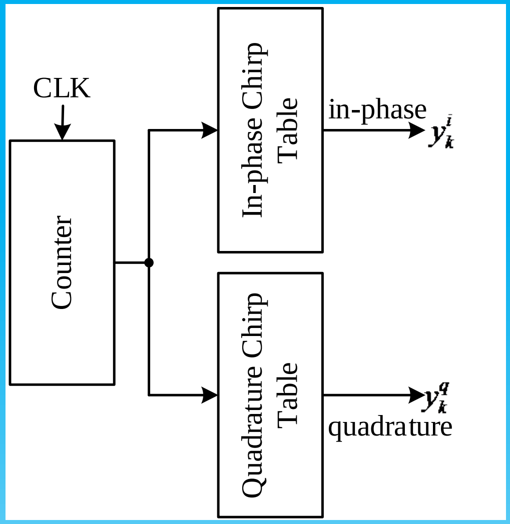


$$y_k^i = d\cos\left(ak^2T_s^2 + bkT_s + c\right)$$

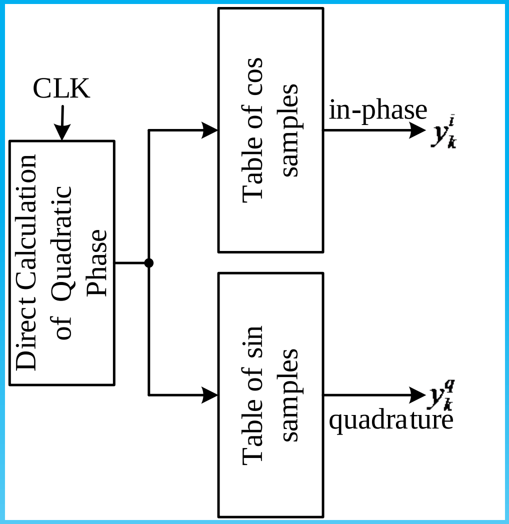
$$y_k = d\exp\left(j\left(ak^2T_s^2 + bkT_s + c\right)\right)$$

$$y_k^q = d\sin\left(ak^2T_s^2 + bkT_s + c\right)$$

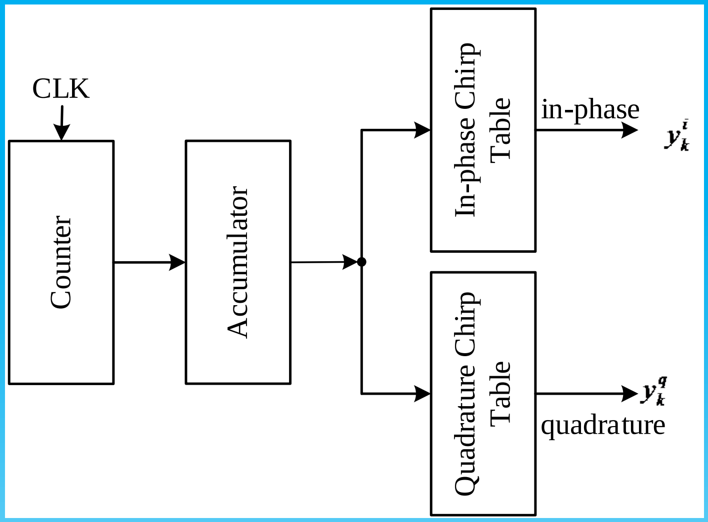
Варианты реализации



Простейшая реализация



Прямое вычисление фазы



Рекурсивное вычисление фазы

Параметры ЛЧМ сигнала

$y_k = d\exp\left(j\left(ak^2T_s^2 + bkT_s + c\right)\right)$

T_s Период дискретизации

$\theta = at^2 + bt + c$ Мгновенная фаза

$\nu = \frac{1}{2\pi}\frac{d\theta}{dt} = \frac{2at+b}{2\pi}$ Мгновенная частота

$\nu_0 = \frac{b}{2\pi}$ Начальное значение мгновенной частоты → $b = 2\pi\nu_0$

$\tau = \kappa T_s$ Длительность сигнала

$\nu_0 + \Delta\nu = \frac{2a\kappa T_s + b}{2\pi}$ Конечное значение мгновенной частоты

$\Delta\nu = \frac{a\kappa T_s}{\pi}$ Девиация → $a = \frac{\pi\Delta\nu}{\kappa T_s}$

$c = 2\pi f_0 T_s$

d

Рекурсивный алгоритм

$k^2 = (k-1)^2 + 2k - 1$

$k = (k-1) + 1$

$$y_k = d\exp\left(j\left(ak^2T_s^2 + bkT_s + c\right)\right) = d\exp\left(j\left(a(k-1)^2T_s^2 + b(k-1)T_s + c\right)\right)\exp\left(j\left(a(2k-1)T_s^2 + bT_s\right)\right) = y_{k-1}w_k,$$

y_{k-1} w_k

$2k-1 = (2k-3) + 2$

$$w_k = \exp\left(j\left(a(2k-1)T_s^2 + bT_s\right)\right) = \exp\left(j\left(a(2k-3)T_s^2 + bT_s\right)\right)\exp\left(j2aT_s^2\right) = w_{k-1}\exp\left\{j2aT_s^2\right\}.$$

w_{k-1}

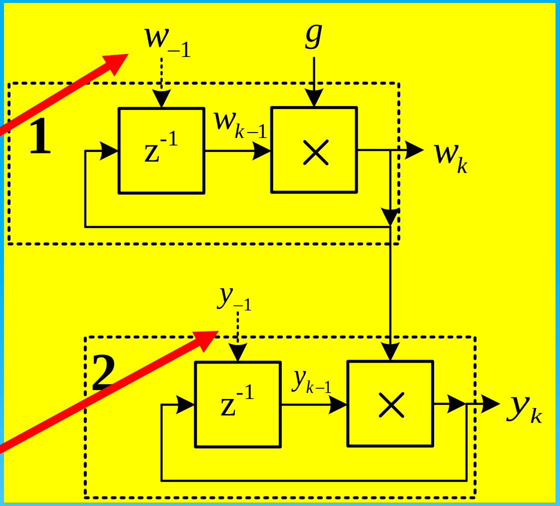
$$\begin{cases} w_k = gw_{k-1}, \\ y_k = w_k y_{k-1}, \end{cases}$$

$k = 0, 1, \ldots, \left\lceil \frac{\tau}{T_s} \right\rceil$

$g = \exp\left(j2aT_s^2\right)$

Реализация предложенного алгоритма в комплексной арифметике

$$\begin{cases} w_k = g w_{k-1}, \\ y_k = w_k y_{k-1}, \end{cases}$$



$$w_k = \exp\left(j\left(a\left(2k-1\right)T_s^2 + bT_s\right)\right)$$

Harmonic signal

$$y_k = d\exp\left(j\left(ak^2T_s^2 + bkT_s + c\right)\right)$$

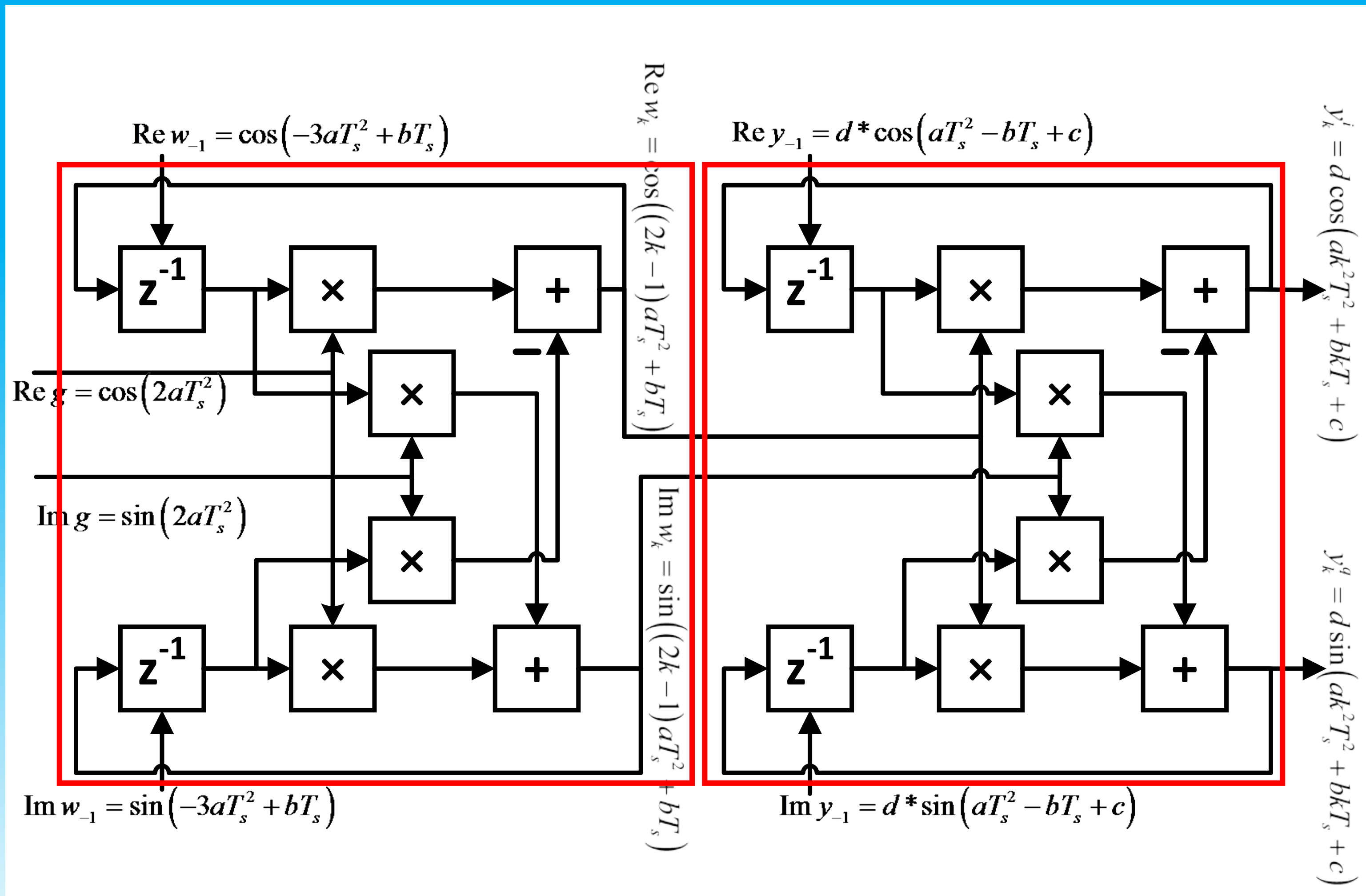
Chirp signal

The initial conditions

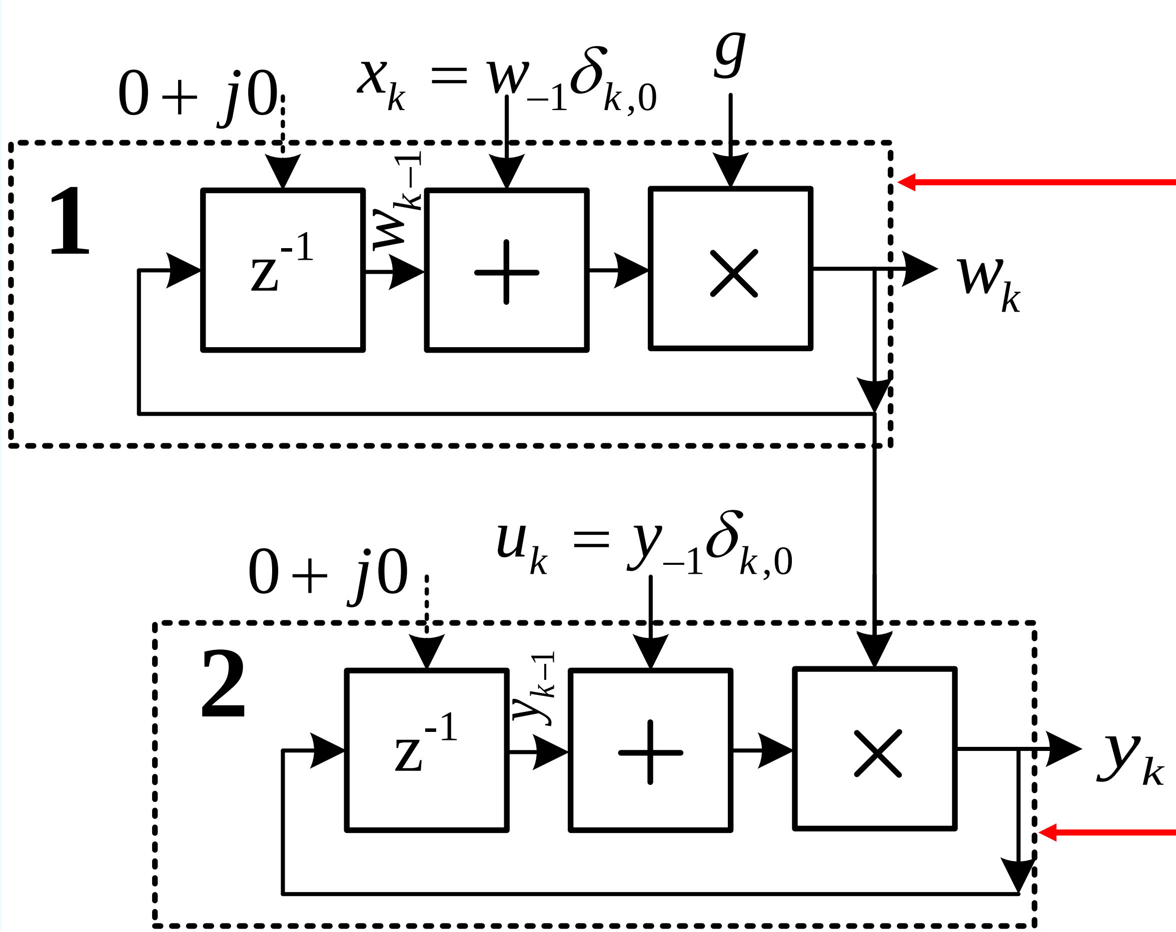
$$w_{-1} = \exp\left(j\left(-3aT_s^2 + bT_s\right)\right)$$

$$y_{-1} = d\exp\left(j\left(aT_s^2 - bT_s + c\right)\right)$$

Реализация предложенного алгоритма в вещественной арифметике

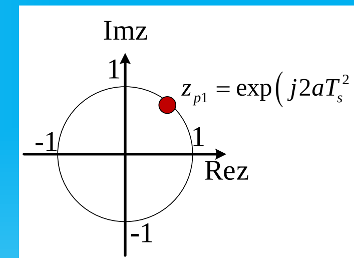


Эквивалентная схема

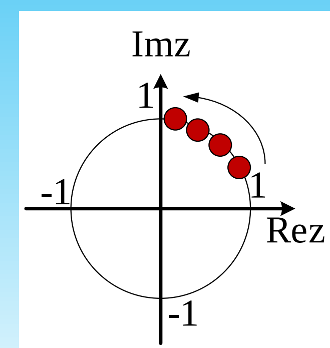


$$H_1(z) = \frac{W(z)}{X(z)} = \frac{1}{\exp(-j2aT_s^2) - z^{-1}} = \frac{\exp(j2aT_s^2)z}{z - \exp(j2aT_s^2)}.$$

$$z_{p1} = \exp(j2aT_s^2)$$



instantaneous pole



Заключение

Предложен рекурсивный когерентный формирователь комплексных отсчетов ЛЧМ сигнала, характеризующийся минимальным объемом памяти, возможностью быстрой перестройки параметров.

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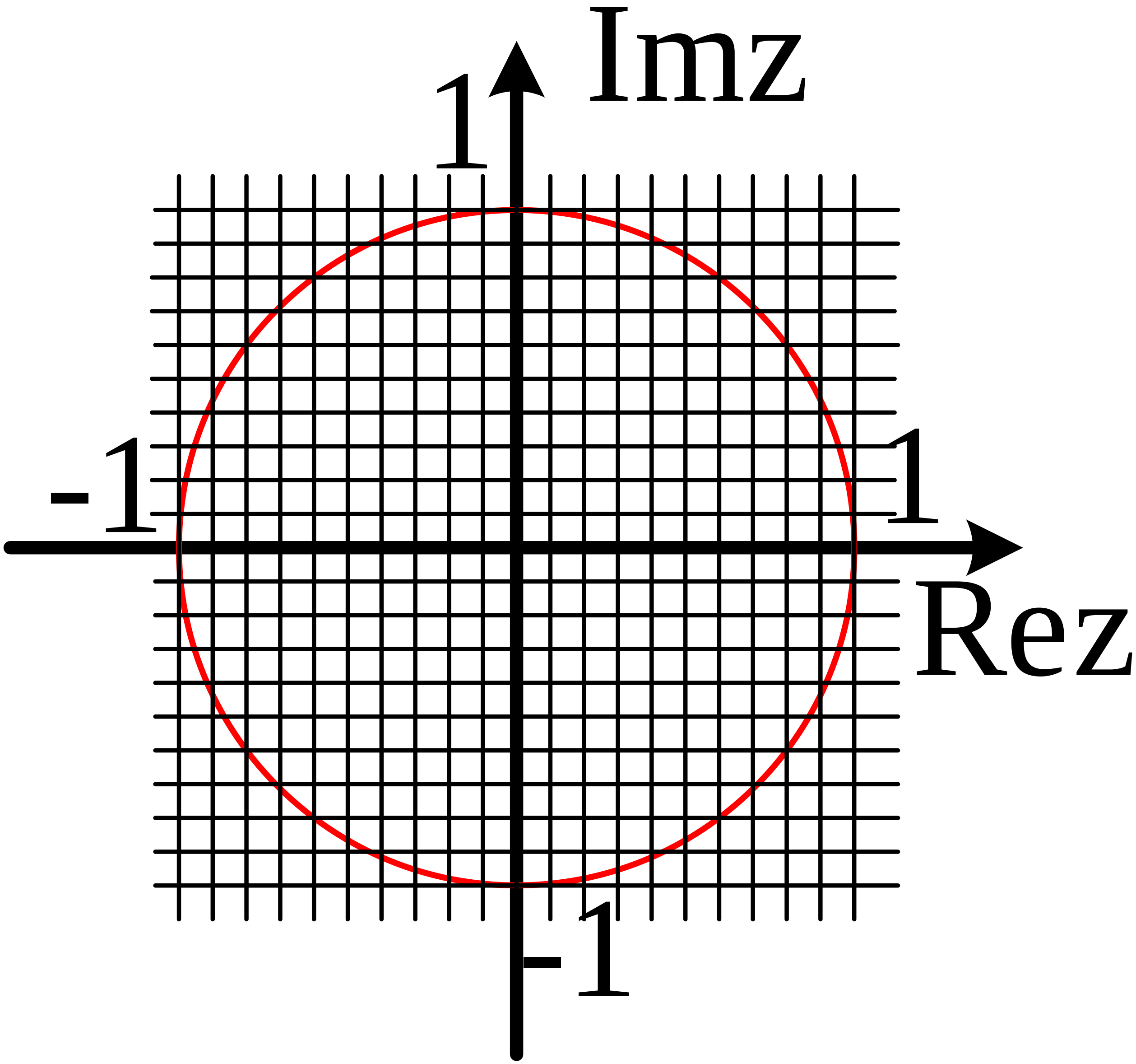
Спасибо за внимание!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