Эквалайзеры с дробной задержкой и обратной связью на базе быстрых RLS-алгоритмов

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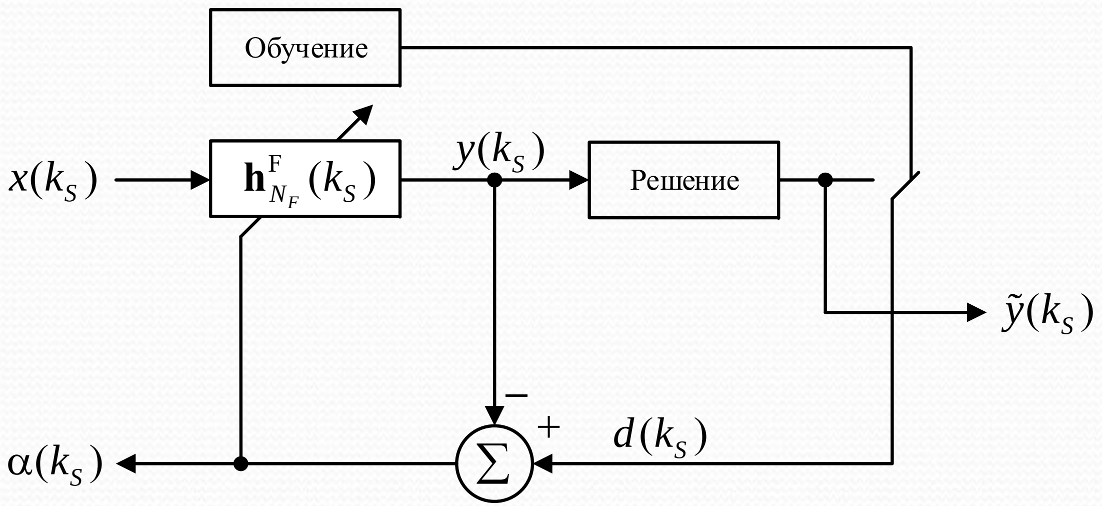
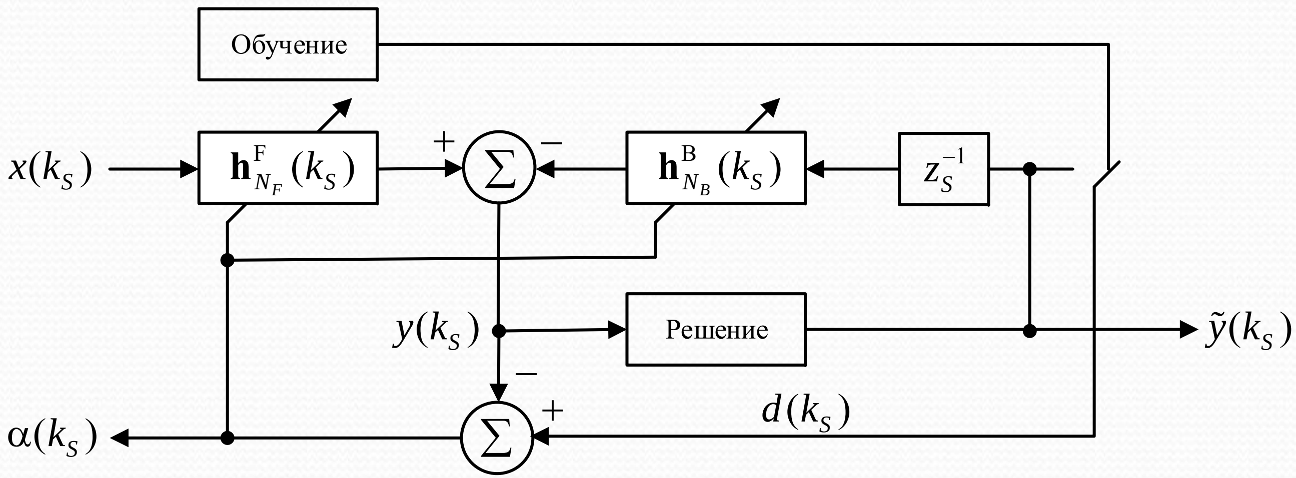
FF и FB эквалайзеры на символьной скорости

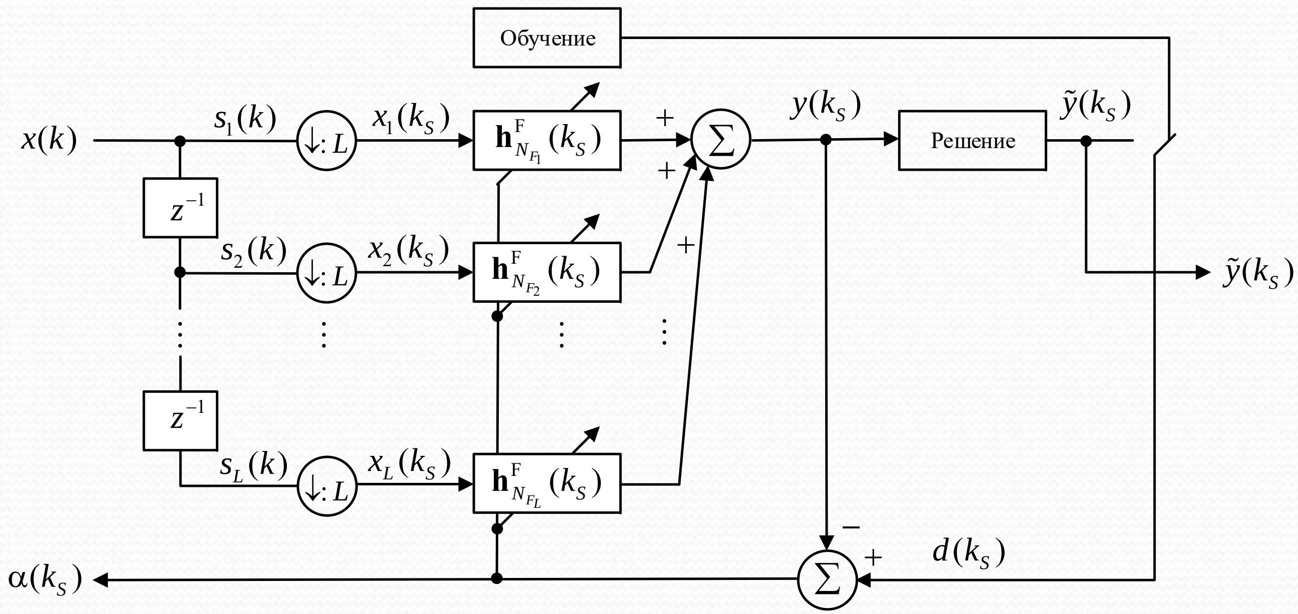


Feed-Forward эквалайзер

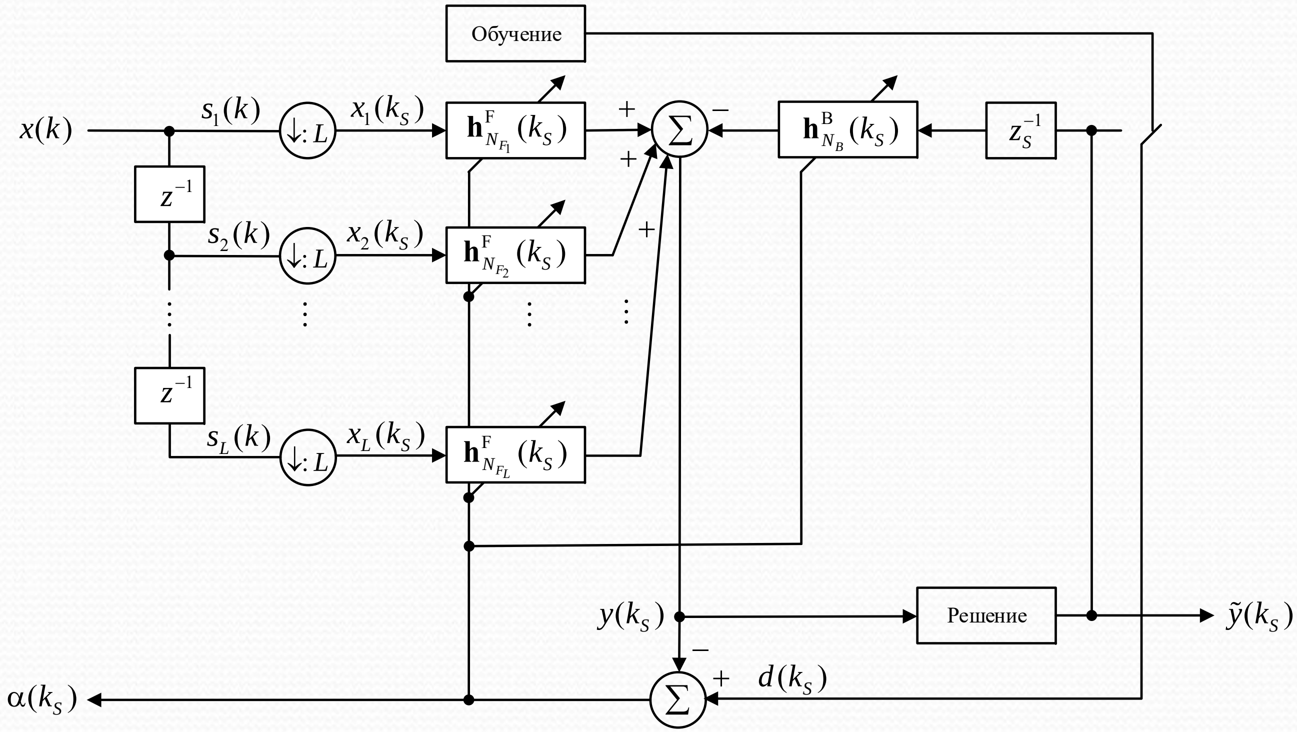


Feed-Backward эквалайзер

FF эквалайзер с дробной задержкой



FB эквалайзер с дробной задержкой



Алгоритм работы адаптивного эквалайзера

В.И. Джиган, «Адаптивная фильтрация сигналов: теория и алгоритмы», МЖ: Техносфера, 2013, 528 с.
Moscow, Russia,
http://www.technosphera.ru/lib/book/333

Initialization : $E_N^{f(m)}(0)=\delta^2;\ \mathbf{h}_N^{f(m)}(0)=\mathbf{0}_N;$
$\mathbf{h}_N^{b(m)}(0)=\mathbf{0}_N;$ **create :** $\mathbf{S}_{N+1}^{(m)}\mathbf{T}_{N+1}^{(m)\mathrm{T}};\ m=1,\ldots,M;\ M=L+1,$
$\mathbf{g}_N^{(M)}(1)=\mathbf{0}_N;\ \mathbf{x}_N^{(0)}(0)=\mathbf{0}_N;\ \mathbf{h}_N(0)=\mathbf{0}_N;\mathbf{s}_L(0)=\mathbf{0}_L;\ k_s=0$

For $k=1,2,\ldots,K$

$\mathbf{s}_L(k)\big|_{2:L}=\mathbf{s}_L(k)\big|_{1:L-1},\ \mathbf{s}_L(k)\big|_1=x(k)$

$l=\mathrm{mod}_L(k)$

if $l=0$

$k_s=k_s+1$

For $l=1,2,\ldots,L$

$\mathbf{x}_{N_{\mathrm{F},l}}^{(l)}(k_s)\big|_{2:N_{\mathrm{F},l}}=\mathbf{x}_{N_{\mathrm{F},l}}^{(l)}(k_s)\big|_{1:N_{\mathrm{F},l}-1},$

$\mathbf{x}_{N_{\mathrm{F},l}}^{(l)}(k)\big|_1=s_l(k)$

End for l

$\mathbf{x}_{N_\mathrm{B}}(k_\mathrm{S})\big|_{2:N_\mathrm{B}}=\mathbf{x}_{N_\mathrm{B}}(k_\mathrm{S})\big|_{1:N_\mathrm{B}-1},\ \mathbf{x}_{N_\mathrm{B}}(k_\mathrm{S})\big|_1=d(k_\mathrm{S}-1)$

$$\mathbf{x}_N^{(0)}(k_\mathrm{S})=\Big[\mathbf{x}_{N_{\mathrm{F},1}}^{(1)\mathrm{T}}(k_\mathrm{S}),\mathbf{x}_{N_{\mathrm{F},2}}^{(2)\mathrm{T}}(k_\mathrm{S}),\ldots,\mathbf{x}_{N_{\mathrm{F},l}}^{(l)\mathrm{T}}(k_\mathrm{S}),\mathbf{x}_{N_{\mathrm{F},l}}^{(l+1)\mathrm{T}}(k_\mathrm{S}),\ldots,\mathbf{x}_{N_{\mathrm{F},L}}^{(L)\mathrm{T}}(k_\mathrm{S}),\mathbf{x}_{N_\mathrm{B}}^{\mathrm{T}}(k_\mathrm{S})\Big]^\mathrm{T}=\Big[\mathbf{x}_{N_1}^{(1)\mathrm{T}}(k_\mathrm{S}),\mathbf{x}_{N_2}^{(2)\mathrm{T}}(k_\mathrm{S}),\ldots,\mathbf{x}_{N_m}^{(m)\mathrm{T}}(k_\mathrm{S}),\mathbf{x}_{N_{m+1}}^{(m+1)\mathrm{T}}(k_\mathrm{S}),\ldots,\mathbf{x}_{N_{M-1}}^{(M-1)\mathrm{T}}(k_\mathrm{S}),\mathbf{x}_{N_M}^{(M)\mathrm{T}}(k_\mathrm{S})\Big]^\mathrm{T}$$

$$\mathbf{x}_N^{(1)}(k_\mathrm{S})=\Big[\mathbf{x}_{N_1}^{(1)\mathrm{T}}(k_\mathrm{S}-1),\mathbf{x}_{N_2}^{(2)\mathrm{T}}(k_\mathrm{S}),\ldots,\mathbf{x}_{N_m}^{(m)\mathrm{T}}(k_\mathrm{S}),\mathbf{x}_{N_{m+1}}^{(m+1)\mathrm{T}}(k_\mathrm{S}),\ldots,\mathbf{x}_{N_{M-1}}^{(M-1)\mathrm{T}}(k_\mathrm{S}),\mathbf{x}_{N_M}^{(M)\mathrm{T}}(k_\mathrm{S})\Big]^\mathrm{T}$$

$\vdots$

$$\mathbf{x}_N^{(m)}(k_\mathrm{S})=\Big[\mathbf{x}_{N_1}^{(1)\mathrm{T}}(k_\mathrm{S}-1),\mathbf{x}_{N_2}^{(2)\mathrm{T}}(k_\mathrm{S}-1),\ldots,\mathbf{x}_{N_m}^{(m)\mathrm{T}}(k_\mathrm{S}-1),\mathbf{x}_{N_{m+1}}^{(m+1)\mathrm{T}}(k_\mathrm{S}),\ldots,\mathbf{x}_{N_{M-1}}^{(M-1)\mathrm{T}}(k_\mathrm{S}),\mathbf{x}_{N_M}^{(M)\mathrm{T}}(k_\mathrm{S})\Big]^\mathrm{T}$$

$$\mathbf{x}_N^{(m+1)}(k_\mathrm{S})=\Big[\mathbf{x}_{N_1}^{(1)\mathrm{T}}(k_\mathrm{S}-1),\mathbf{x}_{N_2}^{(2)\mathrm{T}}(k_\mathrm{S}-1),\ldots,\mathbf{x}_{N_m}^{(m)\mathrm{T}}(k_\mathrm{S}-1),\mathbf{x}_{N_{m+1}}^{(m+1)\mathrm{T}}(k_\mathrm{S}-1),\ldots,\mathbf{x}_{N_{M-1}}^{(M-1)\mathrm{T}}(k_\mathrm{S}),\mathbf{x}_{N_M}^{(M)\mathrm{T}}(k_\mathrm{S})\Big]^\mathrm{T}$$

$\vdots$

$$\mathbf{x}_N^{(M-1)}(k_\mathrm{S})=\Big[\mathbf{x}_{N_1}^{(1)\mathrm{T}}(k_\mathrm{S}-1),\mathbf{x}_{N_2}^{(2)\mathrm{T}}(k_\mathrm{S}-1),\ldots,\mathbf{x}_{N_m}^{(m)\mathrm{T}}(k_\mathrm{S}-1),\mathbf{x}_{N_{m+1}}^{(m+1)\mathrm{T}}(k_\mathrm{S}-1),\ldots,\mathbf{x}_{N_{M-1}}^{(M-1)\mathrm{T}}(k_\mathrm{S}-1),\mathbf{x}_{N_M}^{(M)\mathrm{T}}(k_\mathrm{S})\Big]^\mathrm{T}$$

$$\mathbf{x}_N^{(M)}(k_\mathrm{S})=\Big[\mathbf{x}_{N_1}^{(1)\mathrm{T}}(k_\mathrm{S}-1),\mathbf{x}_{N_2}^{(2)\mathrm{T}}(k_\mathrm{S}-1),\ldots,\mathbf{x}_{N_m}^{(m)\mathrm{T}}(k_\mathrm{S}-1),\mathbf{x}_{N_{m+1}}^{(m+1)\mathrm{T}}(k_\mathrm{S}-1),\ldots,\mathbf{x}_{N_{M-1}}^{(M-1)\mathrm{T}}(k_\mathrm{S}-1),\mathbf{x}_{N_M}^{(M)\mathrm{T}}(k_\mathrm{S}-1)\Big]^\mathrm{T}$$

For $m=M,M-1,\ldots,1$

$\alpha^{f(m)}(k_\mathrm{S})=x_m(k_\mathrm{S})-\mathbf{h}_N^{f(m)\mathrm{H}}(k_\mathrm{S}-1)\mathbf{x}_N^{(m)}(k_\mathrm{S})$

$\alpha^{b(m)}(k_\mathrm{S})=x_m(k_\mathrm{S}-N_m)-\mathbf{h}_N^{b(m)\mathrm{H}}(k_\mathrm{S}-1)\mathbf{x}_N^{(m-1)}(k_\mathrm{S})$

$\mathbf{h}_N^{f(m)}(k_\mathrm{S})=\mathbf{h}_N^{f(m)}(k_\mathrm{S}-1)+\mathbf{g}_N^{(m)}(k_\mathrm{S})\alpha^{f(m)*}(k)$

$e^{f(m)}(k_\mathrm{S})=x_m(k_\mathrm{S})-\mathbf{h}_N^{f(m)\mathrm{H}}(k_\mathrm{S}-1)\mathbf{x}_N^{(m)}(k_\mathrm{S})$

$E^{f(m)}(k_\mathrm{S})=\lambda E^{f(m)}(k_\mathrm{S}-1)+e^{f(m)}(k_\mathrm{S})\alpha^{f(m)*}(k_\mathrm{S})$

$$\bar{\mathbf{g}}_{N+1}^{(m)}(k_\mathrm{S})=\begin{bmatrix}0\\ \mathbf{g}_N^{(m)}(k_\mathrm{S})\end{bmatrix}+\begin{bmatrix}1\\ -\mathbf{h}_N^{f(m)}(k_\mathrm{S})\end{bmatrix}\frac{e^{f(m)}(k_\mathrm{S})}{E^{f(m)}(k_\mathrm{S})}$$

$$\tilde{\mathbf{g}}_{N+1}^{(m)}(k_\mathrm{S})=\mathbf{S}_{N+1}^{(m)}\mathbf{T}_{N+1}^{(m)\mathrm{T}}\bar{\mathbf{g}}_{N+1}^{(m)}(k_\mathrm{S})=\begin{bmatrix}\tilde{\mathbf{q}}_N^{(m)}(k_\mathrm{S})\\ \tilde{q}^{(m)}(k_\mathrm{S})\end{bmatrix}$$

$$\mathbf{g}_N^{(m-1)}(k_\mathrm{S})=\frac{\tilde{\mathbf{q}}_N^{(m)}(k_\mathrm{S})+\mathbf{h}_N^{b(m)}(k_\mathrm{S}-1)\tilde{q}^{(m)}(k_\mathrm{S})}{1-\alpha^{b(m)*}(k_\mathrm{S})\tilde{q}^{(m)}(k_\mathrm{S})}$$

$\mathbf{h}_N^{b(m)}(k_\mathrm{S})=\mathbf{h}_N^{b(m)}(k_\mathrm{S}-1)+\mathbf{g}_N^{(m-1)}(k_\mathrm{S})\alpha^{b(m)*}(k_\mathrm{S})$

End for m

$y(k_\mathrm{S})=\mathbf{h}_N^{\mathrm{H}}(k_\mathrm{S}-1)\mathbf{x}_N^{(0)}(k_\mathrm{S})$

$\alpha(k_\mathrm{S})=d(k_\mathrm{S})-y(k_\mathrm{S})$

$$\mathbf{h}_N(k_\mathrm{S})=\mathbf{h}_N(k_\mathrm{S}-1)+\mathbf{g}_N^{(0)}(k_\mathrm{S})\alpha^*(k_\mathrm{S})=\Big[\mathbf{h}_{N_{\mathrm{F},1}}^{\mathrm{F}(1)\mathrm{T}}(k_\mathrm{S}),\mathbf{h}_{N_{\mathrm{F},2}}^{\mathrm{F}(2)\mathrm{T}}(k_\mathrm{S}),\ldots,\mathbf{h}_{N_{\mathrm{F},l}}^{\mathrm{F}(l)\mathrm{T}}(k_\mathrm{S}),\mathbf{h}_{N_{\mathrm{F},l+1}}^{\mathrm{F}(l+1)\mathrm{T}}(k_\mathrm{S}),\ldots,\mathbf{h}_{N_{\mathrm{F},L}}^{\mathrm{F}(L)\mathrm{T}}(k_\mathrm{S}),\mathbf{h}_{N_\mathrm{B}}^{\mathrm{BT}}(k_\mathrm{S})\Big]^\mathrm{T}$$

$\mathbf{g}_N^{(M+1)}(k_\mathrm{S}+1)=\mathbf{g}_N^{(0)}(k_\mathrm{S})$

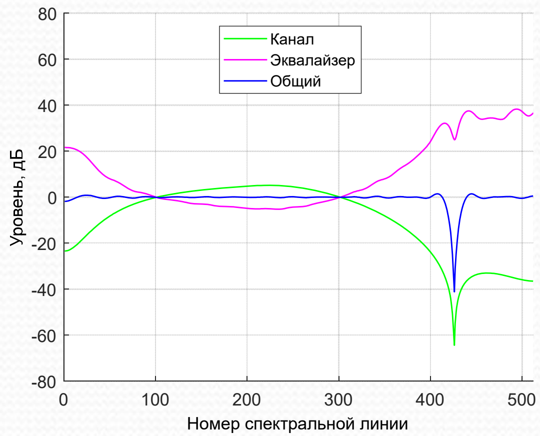
End for if

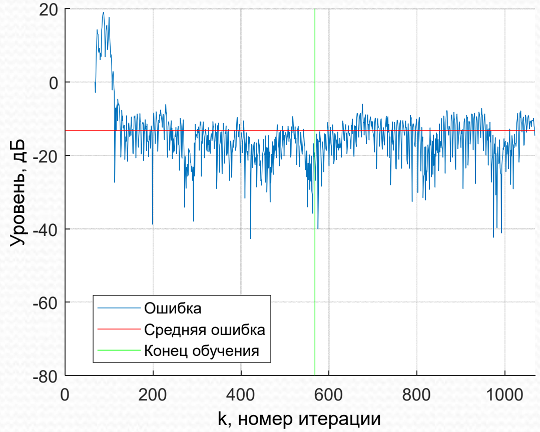
End for k

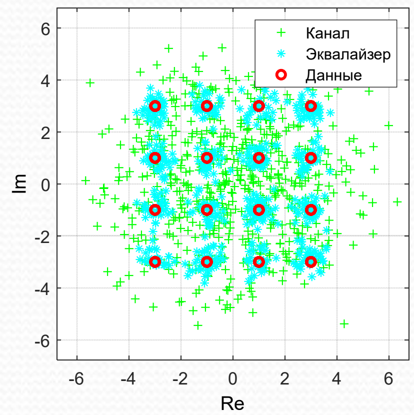
Многоканальный быстрый алгоритм Калмана

Результаты моделирования

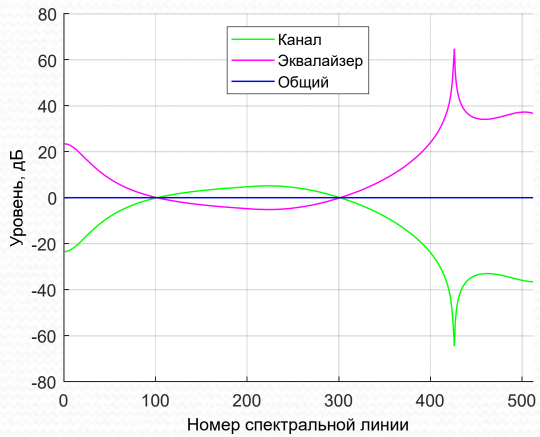
FF эквалайзер

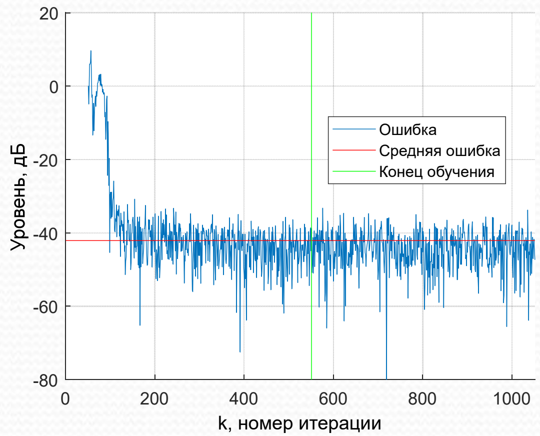


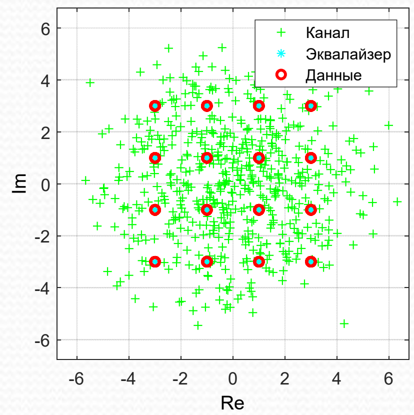




FB эквалайзер





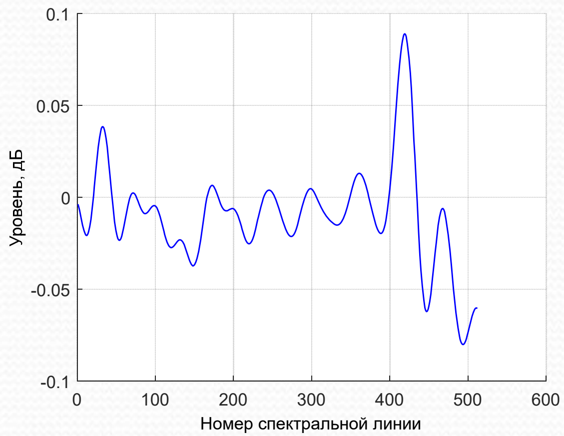


АЧХ

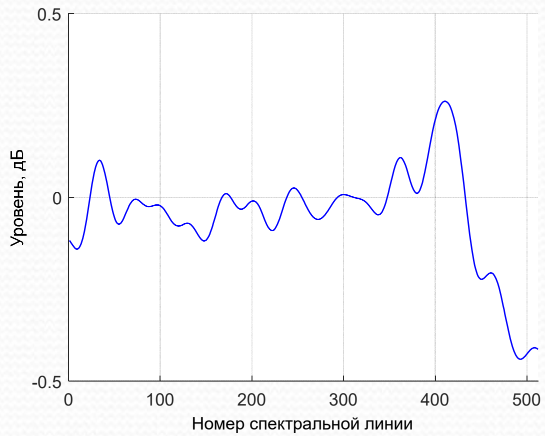
Переходные процессы

Созвездия

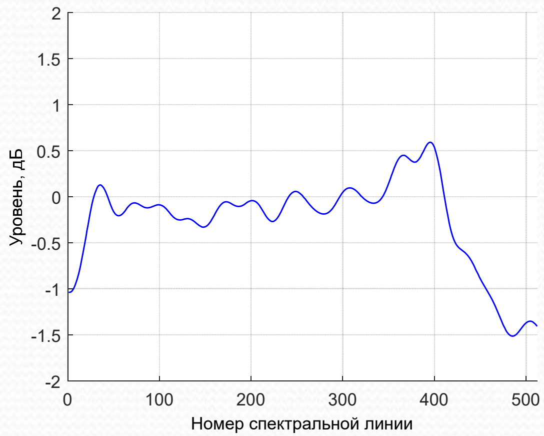
Выровненная АЧХ канала связи



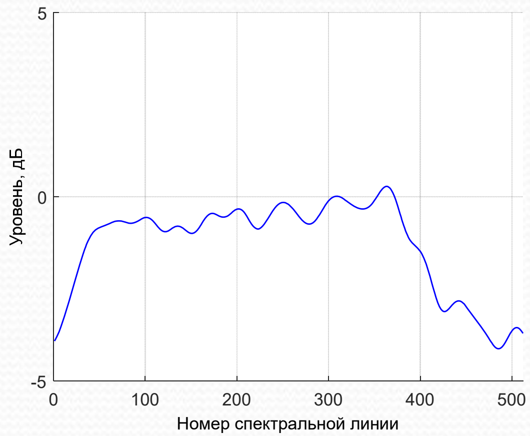
ОСШ=30 дБ



ОСШ=20 дБ



ОСШ=20 дБ



ОСШ=10 дБ

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Вопросы ?

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